



STAMFORD COLLEGE

SCHOOL OF ENGINEERING

FOUNDATION STUDIES IN ENGINEERING (ELECTRICAL AND ELECTRONIC)

KE014: DIFFERENTIAL CALCULUS

Date : 24 May 2006 (Wednesday)

Time : 9.30 am – 11.40 am

Duration: 2 hours + 10 minutes reading time

Instructions to Candidates

1. SIX questions set.
2. Answer any FOUR questions.
3. All questions carry equal marks.
4. Maximum marks attainable: 100

Please ensure that this examination paper contains SIX questions on FIVE printed pages before you start the examination.

Books, papers and other written materials are not allowed to be brought into the examination hall. A candidate who violates the examination rules of Stamford College or commits a malpractice will be disqualified from the examination.

Write your Examination Index Number on each page of your answer booklet.

ANSWER ANY FOUR QUESTIONS**Question 1**

(a) Find the followings:

i) $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1}$ (3 marks)

ii) $\lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$ (3 marks)

iii) $\lim_{x \rightarrow 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x}$ (3 marks)

iv) $\lim_{x \rightarrow -\infty} \frac{2x^2 - 5}{3x^2 + x + 2}$ (3 marks)

v) $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^2 + 2}}{4x + 3}$ (3 marks)

(b) Find the discontinuities (if any) for the following functions:

i) $f(x) = \frac{1}{x-1}$ (2 marks)

ii) $f(x) = \frac{x+2}{x^2 - 3x + 10}$ (3 marks)

(c) Determine whether the following functions are continuous:

i) $f(x) = \sqrt{2x-5} + 3x$, $a=4$ (2 marks)

ii) $f(x) = 3x^2 + 7 - \frac{1}{\sqrt{-x}}$, $a = -2$ (3 marks)

(Total = 25 marks)

Question 2

(a) Find the slope at the given point, P for each of the following functions:

i) $f(x) = -5x^2 + 8x + 2$; P(-1,-11) (3 marks)

ii) $f(x) = x^3 + x$; P(1,2) (3 marks)

iii) $f(x) = 3x^2 - 12x + 8$; P(3,-1) (3 marks)

(b) With the results obtained in part (a)(i), (a)(ii) and a(iii), find an equation of the tangent line for each of the functions in (a)(i), (a)(ii) and a(iii) at their respective P points. (12 marks)

(c) Find the average rate of change for $f(x) = 2x^2$, between $x=0$ and $x=3$.

(4 marks)

(Total = 25 marks)

Question 3

(a) Differentiate the following with respect to x :

i) $f(x) = 5x^2 + 3x$ (2 marks)

ii) $f(x) = \frac{x^3 + 4}{x}$ (2 marks)

iii) $f(x) = 4x^2 - \frac{2}{x} - \frac{3}{x^3}$ (2 marks)

(b) Differentiate the following functions by using the product rule:

i) $f(x) = (x^2 - 4x + 6)(1 - 3x^3)$ (3 marks)

ii) $f(x) = e^x \sin x$ (3 marks)

iii) $f(x) = (x^3 - 3x)(2x^2 + 3x + 5)$ (3 marks)

iv) $f(x) = (3x^3 + 4x)(x - 5)(x + 1)$ (3 marks)

- (c) An object moves in straight line so that its position after t seconds is given by $s = 1 + 2t + 5t^2$. Find the velocity of the object when:
- $t = 0$
 - $t = 2$ and
 - $t = 4$

(7 marks)
(Total = 25 marks)

Question 4

- (a) Solve the following using the chain rule:

i) $\frac{d}{dx} \left[\sqrt{x^3 + \csc x} \right]$ (3 marks)

ii) $\frac{d}{dx} \left[\tan(x^2 + 1) \right]$ (3 marks)

iii) $\frac{d}{dx} \left[x^2(x - 2)^4 \right]$ (3 marks)

- (b) Differentiate the following functions:

i) $f(x) = \frac{3 - 2x - x^2}{x^2 - 1}$ (4 marks)

ii) $f(x) = \frac{x^3}{1 + x^2}$ (4 marks)

iii) $f(x) = \frac{e^x}{\sin x}$ (4 marks)

iv) $f(x) = \frac{(x + 1)(2x - 5)}{x + 2}$ (4 marks)

(Total = 25 marks)

Question 5

(a) By applying L'Hospital's rule, solve the followings:

i) $\lim_{x \rightarrow 0} \frac{\cos x + 2x - 1}{3x}$ (3 marks)

ii) $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{1 - \cos 2x}$ (3 marks)

iii) $\lim_{x \rightarrow (\pi/2)} \frac{4 \tan x}{1 + \sec x}$ (3 marks)

iv) $\lim_{x \rightarrow \infty} \frac{e^{3x}}{x^2}$ (3 marks)

v) $\lim_{x \rightarrow 0} \frac{e^x + e^{-x}}{x^2}$ (3 marks)

(b) Differentiate the following trigonometric functions:

i) $y = \frac{\sin x}{1 + \cos x}$ (3 marks)

ii) $y = \sec x \tan x$ (3 marks)

iii) $y = \sec \theta \cot \theta$ (4 marks)

(Total = 25 marks)

Question 6

(a) Locate the extrema of the following functions with the given intervals:

i) $f(x) = 4 - x^2$; $[-2,1]$ (2 marks)

ii) $f(x) = 1/x^2$; $[-1,2]$ (3 marks)

(b) If $f(x) = x^3 - 12x$, find the maximum and minimum values of f on the closed interval $[-3,5]$ and sketch the graph of f . (5 marks)

(c) If $f(x) = (x-1)^{2/3} + 2$, find the maximum and minimum values of f on the closed interval $[0,9]$ and sketch the graph of f . (5 marks)

(d) Find the critical numbers(if any) of $f(x) = 2x^3 + 3x^2 - 12x$, for intervals $[-\infty, \infty]$ on which f is increasing and decreasing, and locate all relative extrema.

(10 marks)

(Total = 25 marks)

– END OF PAPER –