MATHEMATICS OF FINANCE

Lecture 9
Interest, Present value and Future value

Congratulations!!!

You have won a cash prize! You have two payment options:

A. Receive $10,000 now

OR

B. Receive $10,000 in three years.

Which option would you choose?

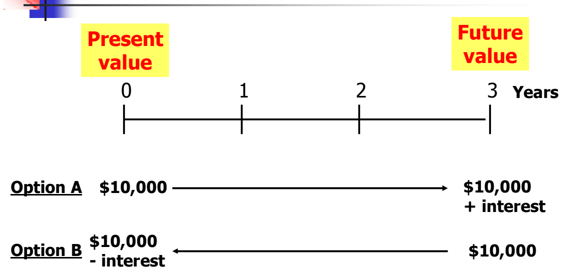
Time Value of Money

An amount received or paid at some time in the future is **not worth as much as** the same amount today.

Time Value of Money

There are a variety of factors which support this observation; risk, inflation, opportunity cost, etc. You probably know that if you leave money sitting idly in your checking account, you are actually losing income you could be earning if you had it in an interest-bearing account. If you let money earn interest, it will grow over time. This is ***the time value of money***.

Time Value of Money



Interest, Present value & Future value

- **9.1 Introduction**
- **9.2 Simple & Compound Interest**
- **9.3 Nominal & Effective Interest Rate**
- **9.4 Present & Future Value**

9.1 Introduction

When does interest incur?

- We borrow *a sum of money* from the lending agent, e.g.: the bank, to pay housing loan
- We deposit *a sum of money* in a savings account, fixed deposits, etc

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How is interest is computed?

Base on the sum of money that is lent or invested over a **period of time, n**.

- This lump sum of money is called the **Principal, P**.

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What is **interest, I**?

It is a fee which is paid for having the use of money.

- It is usually computed as a percentage, called the **interest rate, i**.
- e.g.: 18% per year or 1.5% per month

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In general, if a principal **P** is borrowed at a rate **i**, then after **n** periods of time the borrower will owe the lender an amount, **F** that will include the principal **P** plus the interest **I**.

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Summary of the terms

- Interest, I (*in dollars*)
- Interest rate, i (*in percentage*)
- Principal, P (*in dollars*)
- Number of time periods, n
- Amount or Future Value, F

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9.2 Simple & Compound Interest

When an amount of money is deposited or borrowed over a number of years, the interest earned can be computed with 2 ways:

a) **Simple interest**

b) **Compound interest**

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Differences between Simple & Compound Interest

Simple Interest	Compound Interest
Interest is computed **solely on** the amount of the principal for each of the following periods	Interest earned each period is **added** to the principal for computing interest for the next period
Interest is usually compounded **annually**	Interest is usually compounded **annually**, **semiannually**, **quarterly**, **monthly** or **daily**
I = Pin	I = F - P

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Example 1 (simple interest)

RM100 is invested at 5% simple interest per annum for 5 years. Compute the amount at end of each year.

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Solution (simple interest)

Year	Amount on which interest is calculated	Interest earned at 5%	Accumulated value or future value
1	RM100	RM5	RM105
2	RM100	RM5	RM110
3	RM100	RM5	RM115
4			
5			

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Formula for accumulated value or future value (Simple interest)

F = P+I =P(1+in)

where

F = accumulated value or future value at the end of nth year

P = principle amount or present value

i = interest rate per year

n = number of periods

I = simple interest earned = Pin

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Example 2 (compound interest)

RM100 is compounded annually at 5% for 5 years. Compute the amount at end of each year.

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Solution (compound interest)

Year	Amount on which interest is calculated	Interest earned at 5%	Accumulated value or future value
1	RM100	RM5	RM105
2	RM105	RM5.25	RM110.25
3	RM110.25	RM5.51	RM115.76
4			
5			

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Formula for accumulated value or future value (Compound interest)

$$F = P(1+ i)^n$$

where

F = Accumulated value or future value at the end of n periods

P = Principle amount or present value

n = total number of compounding periods

i = interest rate

NOTE:

In almost all cases, the interest rate in a problem will be stated as

a) an annual interest rate or nominal rate

b) the interest rate per compounding period

When case (a) occurs, the interest rate per compounding period is computed by the formula

Formula for accumulated value or future value (Compound interest)

$$i = \frac{\text{annual nominal rate}}{\text{number of compounding periods per year}}$$

$$i = j_m / m$$

j_m = annual nominal rate

m = number of compounding periods *per year*

NOTE:

For compound interest, the definition of **period** must be the same for both i and n.

For example, if *i* is computed as interest rate per ***quarter***, then n should be representing total number of ***quarters***.

NOTE:

Therefore,

$$n = m \times t$$

where **t** is the number of years and **n** is the total number of compounding periods.

Example 3 (Simple interest)

Find the total amount due on a loan of RM 900 at 16% simple interest at the end of 4 months.

Solution

P = 900
i = 0.16
n = 4/12 year
Find future value, F
F = P(1 + in)
= 900 (1 + 0.16(4/12))
= 948

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Example 4 (Simple interest)

If you can earn an annual rate of 10% on your investments, how much (to the nearest cent) should you pay for a note that will be worth RM 5000 in 8 month?

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Solution

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Example 5 (Simple interest)

If you pay RM750 for a note that will be worth RM1200 in 15 months, what annual simple interest rate will you earn ?

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Solution

P = 750
F = 1200
n = 15/12 =1.25
Find i
F = P(1 + in)
1200 = 750(1+ i 1.25)
i = 0.48
= 48%

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Example 6 (Compound interest)

If RM 1000 is invested at 8% compounded
(a) annually
(b) semiannually
(c) quarterly
(d) monthly
What is the accumulated amount after 5 years? Write your answers to the nearest cents

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Solution

(a) $i = j_m/m = j_1/1 = 0.08/1 = 0.08$

1 year = 1 time of compounding

5 years = 5 x 1

therefore, n = 5

$F = P(1+i)^n$

$F = 1000(1 + 0.08)^5 = 1469.33$

Note: compound interest earned,

I = F – P = 469.33

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(b) $i = j_m/m = j_2/2 = 0.08/2 = 0.04$

1 year = 2 times of compounding

5 years = 5 x 2

therefore, n = 10

$F = P(1+i)^n$

$F = 1000(1 + 0.04)^{10} = 1480.24$

Note: compound interest earned,

I = F – P = 480.24

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NOTE:

Frequency	Interest
Annually	469.33
Semiannually	480.24
Quarterly	485.95
Monthly	489.85

Compounding more frequent will get more pays!

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Example 7

How much should you invest now at 10% compounded quarterly to have RM8000 toward the purchase of a car in 7 years?

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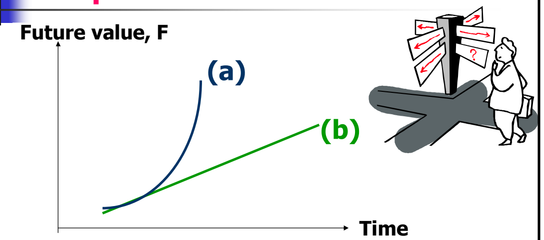
Solution

The effect of simple interest and compound interest



(a) the effect of compound interest
(b) the effect of simple interest

9.3 Nominal and Effective Interest rate

Which investment is better?

12% compounded monthly

Investment A

12.2% compounded semiannually

Investment B

- In normal practices in business, the given annual rate of interest is called nominal rate, j_m
- However, compounding may be semiannually, quarterly, monthly or daily

- compounding more frequent will get more pays , for example, refer to the table below:

One year balances at various compounding periods for RM 100 initial deposit and j_m=10%

Compounded	Finance balance
Annually	110.0
Semiannually	110.25
Quarterly	110.38
Monthly	
Daily	

- People need to know how much more semiannually, quarterly or monthly compounding pays than annually compounding at nominal rate
- It led to the idea of an *effective interest rate,r_e* (Actual percentage rate)

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- the effective interest rate is always greater than nominal rate
- The effective interest rate can be calculated for any compounding period by using the formula below:

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$$r_e = (1 + i)^m - 1$$

where

$i = j_m/m$

m= number of compounding period in one year

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9.3 Nominal and Effective Interest rate

Which investment is better?

$r_e = (1 + 0.12/12)^{12} - 1$
$= 0.1268$
$= 12.68\%$

Investment A

$r_e = (1 + 0.122/2)^2 - 1$
$= 0.1257$
$= 12.57\%$

Investment B

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Changes in the effect of compounding at different rates

Case	Nominal rate, j_m	r_e for monthly compounding	Effective increase
I	6%	6.17%	0.17%
II	12%	12.68%	0.68%
III	18%		

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- So, investors get to know how much more pays they can obtained if the interest is compounded monthly
- For example, in case I, investor will earn 0.17% more if the interest is compounded monthly

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Example 8

A saving and loan pays 7% compounded quarterly . What is the effective interest rate? (express the answer as a percentage)

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Solution

$r_e = (1 + i)^m - 1$

$= (1 + 0.07/4)^4 - 1$

$= 0.0719$

$= 7.19\%$

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Example 9

A saving and loan want to offer a CD (certificate of deposit) with a monthly compounding rate that has an effective rate of 7.5%. What annual nominal rate compounded monthly should they use ?

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Solution

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Example 10

A saving and loan association offers guaranteed investment certificates paying interest at j_{12}=11.25%, j_2=11.5%, j_1 =11.75%. Which option is the best ?

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Solution:

for j_{12} =11.25%

$r_e = (1 + 0.1125/12)^{12} - 1$

$= 0.1185$

for j_2 =11.5%

$r_e = (1 + 0.115/2)^2 - 1$

$= 0.1183$

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Solution:

for j_1= 0.1175

$r_e = (1+0.1175/1)^1 - 1$

$= 0.1175$

therefore

at j_{12} =11.25% where r_e= 0.1185 (highest value), give the best rate of return

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Solution:

Note:

For annually compounded interest rate,

$$i = j_m = r_e$$

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9.4 Present and Future Value

Suppose that $100 is deposited in a savings account that pays 6% compounded annually. Then at the end of two years, the account is worth

$$100\,(1.06)^2 = 112.36$$

The compound amount of $112.36 is the ***future value*** of the $100. $100 is the ***present value*** of the $112.36.

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Present and Future Value

From compound amount formula,

$$\mathbf{F = P\,(1 + i)^n}$$

$$\mathbf{P = F\,(1 + i)^{-n}}$$

where P is called the Present value and F is called the Future value

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Example 11

Find the present value of $1000 due after three years if the interest rate is 9% compounded monthly.

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Solution:

F = $1000

i = 0.09/12 = 0.0075

n = 12 x 3 = 36

$P = 1000\,(1.0075)^{-36}$

$= \$764.15$

This means that $764.15 must be invested at 9% compounded monthly to have $1000 in three years.

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Example 12

A trust fund for a child's education is being set up by a single payment so that at the end of 15 years there will be $50,000. If the fund earns interest at the rate of 7% compounded semiannually, how much money should be paid into the fund?

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Solution:

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