CHAPTER NINE : THE NORMAL DISTRIBUTION

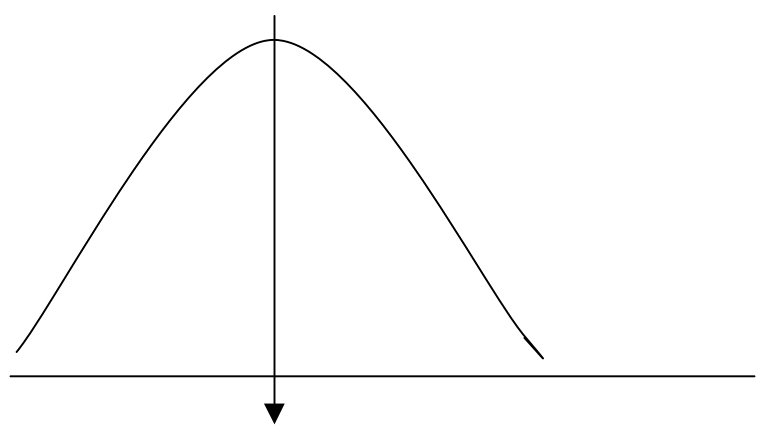
Introduction

- The normal distribution is one of the most important continuous probability distributions. The normal distribution comes close to fitting the actual observed frequency distributions of many phenomena including human characteristics such as weights, heights, etc.

Characteristics of a Normal Distribution

The Normal distribution has the following characteristics:

- The graph of a normal distribution is called the normal curve. The normal curve is continuous, bell-shaped and is symmetrical about the mean (μ).
- The two ends of the normal curve are asymptotic to the x-axis, i.e., they come closer and closer to the horizontal axis without ever touching it.
- The mean, median and mode are located at the same point, i.e., at $x = \mu$.
- The total area under the curve is equal to 1. The ordinate at the mean of the distribution divides the total area under the normal curve into two equal parts.



Mean = Median = Mode

The *z* score

- The normal curve is completely determined by its mean and standard deviation.
- As the area under the normal curve depends on its mean (μ) and the standard deviation (σ), different values of mean and standard deviation would result in different curves hence different areas.

- To overcome the problem, we convert the variable *x* in terms of standard units with mean zero ($\mu = 0$) and standard deviation of unity ($\sigma = 1$).

  The *z* score is defined as $Z = \dfrac{x - v}{\sigma}$
- The number of standard deviation of any observation is from the mean of the distribution.

Example 1

Assume we have a normal distribution with $\mu = 120$ and $\sigma = 10$. For each of the following, determine the area covered under the normal curve.

a. x < 108
b. x > 108
c. 113 < x < 126

Solution

a. x < 108

$$Z = \frac{108 - 120}{10}$$
$$= \underline{-1.2}$$

Thus, Pr (z < -1.2) = Pr (z >1.2)
=0.5 – 0.3849 = <u>0.1151</u>

b. x > 108

$$Z = \frac{108 - 120}{10}$$
$$= \underline{-1.2}$$

Thus, Pr (z > -1.2) = Pr (z <1.2)
=0.5 + 0.3849 = <u>0.8849</u>

c. x > 113

$$Z = \frac{113 - 120}{10}$$
$$= \underline{-0.7}$$

Thus, Pr (z < -0.7) = Pr (z >0.7)
=0.5 + 0.2580
= <u>0.7580</u>

x < 126

$$Z = \frac{126 - 120}{10}$$
$$\underline{=0.6}$$

Thus, Pr (z < 0.6) = 0.5 + 0.2258
= <u>0.7258</u>

Thus, Pr (-0.7 < z < 0.6) = Pr (z < 0.6) - Pr (z > -0.7)
= Pr (z < 0.6) –[1 - Pr (z < 0.7)]
= 0.7258 – (1 – 0.7580) = 0.4838

Example 2

The time taken to complete a particular type of job is known to be normally distributed with mean 6.4 hours and standard deviation 1.2 hours. What is the probability that a randomly selected job of this type takes:

a. less than 6 hours
b. less than 7 hours
c. between 6 and 7 hours

Solution

a. Standardizing 7 gives $Z = \frac{7 - 6.4}{1.2} = 0.5$

We require Pr (z < - 0.5)
From the table, Pr (0 < z < 0.5) = 0.1915
Thus, Pr (z < 0.5) = 0.5 + 0.1915 = 0.6915

$\therefore$ The probability that a randomly selected job takes less than 7 hours is 0.6915.

b. Standardizing 6 gives $Z = \frac{6 - 6.4}{1.2} = -0.33$

We require Pr (z < - 0.33) = Pr (z > 0.33)
From the table, Pr (0 < z < 0. 33) = 0.1293
Thus, Pr (z > 0.5) = 0.5 - 0.1293 = 0.3707

$\therefore$ The probability that a randomly selected job takes less than 6 hours is 0.3707.

c. Probability that time taken to complete job lies between 6 and 7, Pr (6 < T < 7)

= Pr (T < 7) – Pr (T < 6)
= 0.6915 – 0.3707 = 0.3208

$\therefore$ The probability that a randomly selected job takes between 6 and 7 hours is 0.3208.

Example 3

Company records show that the weekly distance traveled by their salesmen is approximately normally distributed with mean 800 miles and standard deviation 90 miles. The sales manager considers that salesmen who travel less than 600 miles in one week are performing poorly.

a. If the company employs 200 salesmen, how many would be expected to perform poorly in a particular week?

b. The sales manager wishes to identify the number of miles traveled in one week, above which only 1% of salesmen are expected to exceed. What weekly mileage is this?

Solution

a. Standardizing 600 gives $Z = \frac{600-800}{90} = -2.22$

We require Pr (z < -2.2) = Pr (z > 2.22)
From the table, Pr (0 < z < 2.22) = 0.4868
Thus, Pr (z > 2.22) = 0.5 - 0.4868 = <u>0.0132</u>

$\therefore$ We would expect (300 X 0.0132 = 2.64) $\cong$ 3 salesmen to perform poorly in any one week.

b. If 1% (0.01) of salesmen are expected to exceed x miles per week, it must be 99% (0.99) of salesmen will travel less than x miles per week.

From the table, if the area under the curve is 0.49, then z = 2.34

$\Rightarrow \quad \frac{x-800}{90} = 2.34$

x – 800 = 2.34 (90)
x = 210.6 + 800 = 1010.6

$\therefore$ 1011 miles is the distance traveled weekly that only 1% of salesmen will exceed.

Learning Outcomes

- Students should be able to understand normal distribution characteristics.
- Students should be able to estimate the *z* score.

Basic Reading

1. Saravanan Kullandavelli (1994) LCCI Business Statistics; 5th ed. Malaysia; Stamford College Group Publishing.
2. A Francis (1995) Business Mathematics and Statistics; 4th ed. London DP Publications Ltd.

Revision Questions

1. Assume we have a normal distribution with μ = 120 and σ= 10. For each of the following determine the area covered under the normal curve.
   a. $x < 108$
   b. $x > 108$
   c. $113 < x < 126$

2. The time taken to complete jobs of a particular type is known to be normally distributed with mean 6.4 hours and standard deviation 1.2 hours. What is the probability that a randomly selected job of this type takes:
   a. less than 7 hours
   b. less than 6 hours
   c. between 6 and 7 hours

3. The duration of a ferry crossing (outward journey) is normally distributed with a mean of 50 minutes and a standard deviation of 4 minutes. Calculate the probability that a randomly selected crossing lasts:
   a. more than 56 minutes
   b. between 48 and 54 minutes.