CHAPTER EIGHT: PROBABILITY

Introduction

- Probability is the chance of something happening. It can be defined as follows:

  "The probability, P(A), that an event A will occur is the number of equally likely outcomes in which A occurs divided by the total number of outcomes". That is,

$$P(A) = \frac{\text{Number of equally likely outcomes for A}}{\text{Total number of outcomes}}$$

Example 1

A letter is chosen at random from the word 'STATISTICS'. Find the probability that it is: a) A b) S

Solution

The total number of outcomes, E = 10

a) Probability of getting a 'A'

The total number of letter 'A' in the word = 1

So, $P(A) = \frac{1}{10} = 0.1$

b) Probability of getting a 'S'

The total number of letter 'S' in the word = 3

So, $P(A) = \frac{3}{10} = 0.3$

Types of Events

- Events can be categorized into 4:

a) Independent Events

Two events A and B are said to be independent if the occurrence or non-occurrence of one event does not affect the occurrence or non-occurrence of the other event. Example of such an event would be throwing a die two times. Whatever number is produced in the first throw does not affect whatever number that would be produced on the second throw.

b) Mutually Exclusive Events

Two events A and B are said to be mutually exclusive if the occurrence of one event excludes the occurrence of the other event and vice-versa. Example of such an event would be the throwing of a coin. It is not possible to get a head and a tail at the same time.

c) Mutually non-exclusive events

Two events A and B are said to be mutually non-exclusive events if the probability of both the events A and B happening together is not equal to zero. In other words, both the events A and B have a chance of happening together.

d) Dependent Events

An event A is said to be dependent on another event B if the probability of event A occurring depends on whether event B has occurred or not.

Laws of Probability

a) Non-Negativity Law

- If A is an event and P(A) is the probability of event A happening, then

$$0 \leq P(A) \leq 1$$

b) Complement Law

- The complement of event A happening is event A not happening which is denoted as $\overline{A}$. The probability of event A not happening is given as:

$$P\left(\overline{A}\right) = 1 - P(A)$$

Example 2

If the probability of getting an event A is 0.3, then the probability of NOT getting the event A is:

$$P\left(\overline{A}\right) = 1 - P(A)$$
$$P\left(\overline{A}\right) = 1 - 0.3$$
$$= \underline{0.7}$$

c) Additive Law

- The additive law of probability states that if A and B are two events, then the probability of event A OR event B happening is given as follows:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Where P(A or B) = probability of event A OR event B happening
P(A) = probability of event A happening
And P(B) = probability of event B happening

- If both the events A and B are mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B)$$

d) Multiplicative Law

- The multiplicative law of probability states that if A and B are two independent events then the probability of both events A and B happening together is equal to the product of the probabilities of the two events.

$$P(A/B) = \frac{P(A \text{ and } B)}{P(B)}$$

Where P(A/B) is read as "the probability of event A occurring knowing that event B has already occurred.

- If A and B are independent, then

$$P(A/B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A).P(B)}{P(B)} = P(A)$$

Applications of the Laws of Probability

Example 3

Two computers, A and B, operate independently and their probabilities of being out of order are 0.02 and 0.05, respectively. Find the probabilities of the following:

a) Both computers are out of order
b) Both computers are operational
c) Computer A is operational but B is out of order
d) Computer B is operational but A is out of order
e) At least one computer is out of order

Solution

Let's say the probability of A being out of order is P(A) = 0.02. So, the probability of A being operational would be $P(\overline{A})$ and is computed as follows:

$$P(\overline{A}) = 1 - P(A) = 1 - 0.02 = 0.98$$

Let's say the probability of B being out of order is P(B) = 0.05. So, the probability of B being operational would be $P(\overline{B})$ and is computed as follows:

$$P(\overline{B}) = 1 - P(B) = 1 - 0.05 = 0.95$$

a) P (Both out of order)

= Probability of both A and B out of order
= P(A) and P(B)
= 0.02 x 0.05
= 0.001

b) P (Both are operational)

= Probability of both A and B operational
= $P(\overline{A})$ and $P(\overline{B})$
= 0.98 x 0.95
= 0.931

c) P (Computer A is operational but B is out of order)

= P(A operational) AND P(B out of order)
= $P(\overline{A})$ and P(B)
= 0.98 x 0.05
= 0.049

d) P (Computer B is operational but A is out of order)

= P(B operational) AND P(A out of order)
= $P(\overline{B})$ and P(A)
= 0.95 x 0.02
= 0.019

e) <u>P (At least one computer is out of order)</u>

Since both the computers operate independently, four combinations are possible:

Condition 1: Both out of order OR
Condition 2: A operational, B out of order OR
Condition 3: A out of order, B operational OR
Condition 4: Both operational

'At least one computer is out of order' means that any one or both the computers are out of order. This is indicated by combination of 1 or 2 or 3.

The probability of 'At least one computer is out of order' can be calculated using 2 approaches.

<u>Approach 1</u>

= P (At least one computer is out of order)
= P(A). P(B) or P $\left(\overline{A}\right).P\left(B\right)$ or P(A). P $\left(\overline{B}\right)$
= 0.001 + 0.049 + 0.019
= <u>0.069</u>

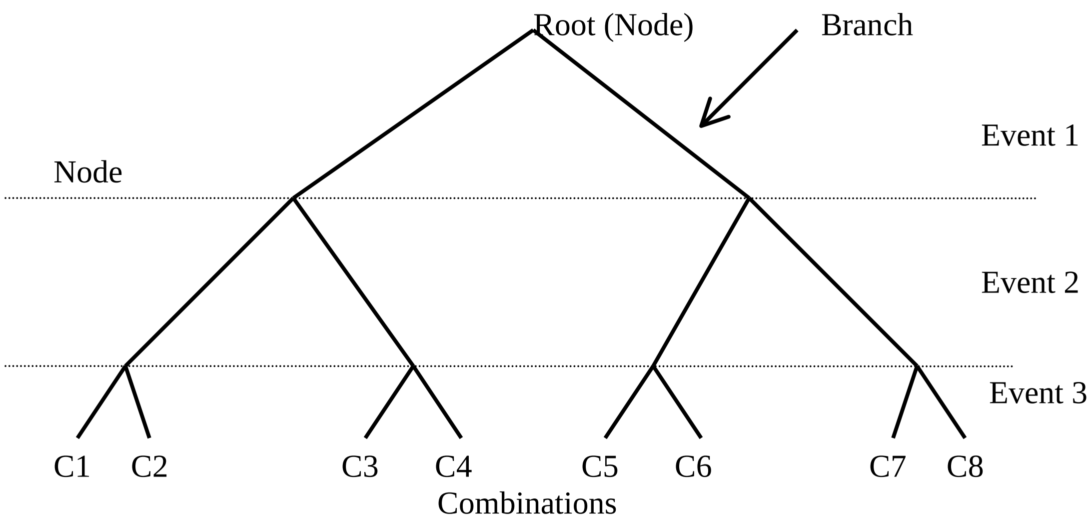
<u>Approach 2</u>

= P (At least one out of order)
= 1-P (Both operational)
= 1 - $\left(P\left(\overline{A}\right).P\left(\overline{B}\right)\right)$
= 1 – (0.98 x 0.95)
= 1 – 0.931
= <u>0.069</u>

<u>Probability Tree</u>

- One of the major problems in probabilities is to ensure that all possible combinations are considered. A probability tree is a diagrammatic approach to indicate all possible combinations of the problem.

- A typical probability tree might look as shown below:



- Every tree will start a root which is also a node. The starting point of a node indicates the starting of an event. Every node has 2 branches: the success branch and the failure branch. All possible paths for branches give the combinations (e.g.; C1, C2, ….., C8).

Example 4

A manufacturer assembles a toy from three independently produced components, each of which has a probability of 0.01 of being defective. What is the probability of:

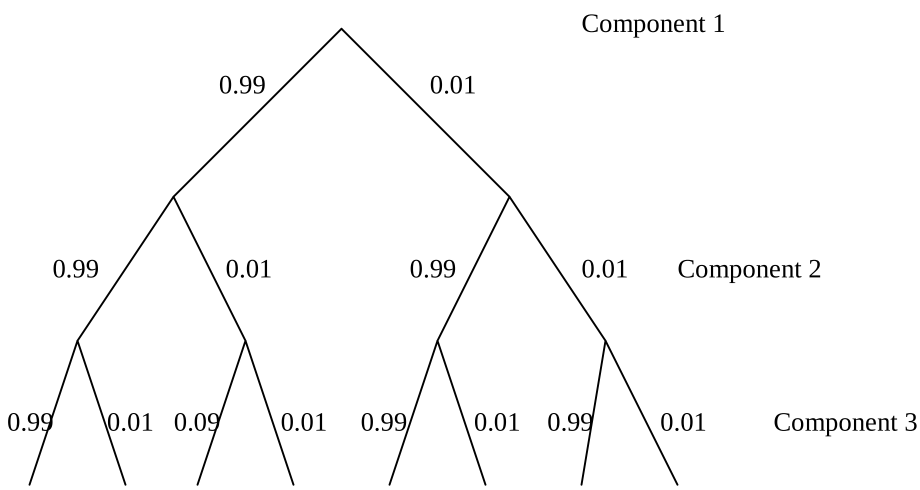
a) no toys are defective
b) a toy being defective

Solution

Let's say the probability of a component being defective is P (C) = 0.01. So, the probability of the component being not defective would be:

$$P\left(\overline{C}\right) = 1 - P(C)$$
$$= 1 - 0.01$$
$$= 0.99$$

The probability tree can be drawn as follows:



a) <u>P (No toys are defective)</u>

P (No defect) = 0.99 x 0.99 x 0.99
= <u>0.9703</u>

b) <u>P (one toy defective)</u>

P (A defect) = 1 – P (No defect)
= 1 – 0.9703
= <u>0.0297</u>

Learning Outcomes

- Students should be able to use probability to calculate the chance of something to happen.
- Students should be able to know the four category of probability events.
- Students should be able to calculate probability using the law of probability.

<u>Basic Reading</u>

1. Saravanan Kullandavelli (1994) <u>LCCI Business Statistics;</u> 5th ed. Malaysia; Stamford College Group Publishing.
2. A. Francis (1995) <u>Business Mathematics and Statistics;</u> 4th ed. London DP Publications Ltd.

Revision Questions

1. Candidates in a Statistics examination may either fail, pass, obtain a credit or obtain a distinction with probabilities of 0.1, 0.6, 0.2 and 0.1, respectively. What are the probabilities of a candidate:

   a) Not failing at all
   b) Either obtaining a credit or a distinction
   c) Failing to obtain either a credit or distinction

2. A die is thrown two times. Calculate the probability:

   a) of getting a "1" in the first throw and "5" in the second
   b) that the total of the result of the first throw and the second is 9

3. In a study of 100 women, 40 had children, 25 had a job and 15 had both children and job. What was the probability that a randomly chosen women had:

   a) Both a job and children
   b) Neither children nor job
   c) Had a job or had children