
CHAPTER THREE: MEASURES OF LOCATION (AVERAGE)**Introduction**

- In any given distribution, there is a value around which the other values lie. This value is called the average. Since the average is the point of location around which the data values cluster, it is commonly referred to as a **measure of location**.

Arithmetic mean**Advantages:**

1. It can easily be understood by most people,
2. It takes into account every individual item,
3. It can be used for further statistical analysis.

Disadvantages:

1. It can easily be influenced by extreme values,
2. It may not correspond with an actual item.

Median and Quartiles:**i) Q₁ (First or lower quartile)**

The first quartile is the item that is at the $\frac{1}{4}$ th position of the data set.

ii) Q₂ (Median)

The median is the item that is at the $\frac{1}{2}$ th position of the data set.

iii) Q₃ (Third or upper quartile)

The third quartile is the item that is at the $\frac{3}{4}$ th position of the data set.

Advantages:

1. Only the value of the middle item need to be determined,
2. It is not easily affected by extreme value.

Disadvantages:

1. It requires the arranging of data in ascending order which can be tedious,
2. It is not suitable for further statistical analysis.

Mode

For a given set of data, the mode is the item that occurs most frequently in the set.

Advantages:

1. It is easily determined without much mathematical calculation,
2. It is not essential to know all the items in the series to compute the mode,
3. It is not affected by extreme values in the data,
4. It may be applied to quantitative as well as qualitative data.

Disadvantages:

1. It does not take every single data into consideration,
2. It becomes less useful as an average when the distribution has more than a single mode,
3. The mode cannot be subjected to further mathematical treatment.

A. Computing the Average for the Set of Data Items**Arithmetic Mean**

- Formulae:

$$\text{Mean, } \bar{x} = \frac{\sum X}{N}$$

Example 1: Given the following set of data items, compute the arithmetic mean:
2, 3, 1, 0, -2, 5

Solution

$$\text{Mean, } \bar{x} = \frac{\sum X}{N}$$

$$\text{Where } \sum X = 2 + 3 + 1 + 0 - 2 + 5 = 9$$

$$\text{And } N = 6$$

$$\text{So, } \bar{x} = \frac{\sum X}{N} = \frac{9}{6} = 1.5$$

Example 2: Given the following data set, compute the arithmetic mean:
-3.2, 2.7, 1.4, 0.8, 5.6, 2.3

Solution

$$\text{Mean, } \bar{x} = \frac{\sum X}{N} = \frac{-3.2 + 2.7 + 1.4 + 0.8 + 2.3}{6} = \frac{9.6}{6} = 1.6$$

Median

Approach: Arrange the values in order of magnitude, lowest to highest, and select as the average the one that occurs in the middle of the list.

Example 3: Given the following set of data, determine the median.

128, 32, 64, 256, 96

Solution

Step 1: Arrange the set in ascending order: 32, 64, 96, 128, 256

Step 2: Determine number of items: $N = 5$

Step 3: Median = $\frac{1}{2}(N+1)^{\text{th}}$ position = $\frac{1}{2}(5+1)^{\text{th}}$ position = 3^{rd} position

Step 4: Identify the 3^{rd} position from the sorted list in Step 1. The value is 96.

Hence, Median = 96

Note:

If N is an EVEN number

⇒ 2 positions are identified;

⇒ Average of the values at these two positions is the median.

Example 4: Given 9 5 2 1 0 8 4 3. Determine the median.

Solution

Step 1: 0 1 2 3 4 5 8 9

Step 2: $N = 8$

Step 3: Median = $\frac{1}{2}(N+1)^{\text{th}}$ position = $\frac{1}{2}(8+1)^{\text{th}}$ position = 4.5^{th} position

Step 4: Since 4.5^{th} position is between the 4^{th} and 5^{th} positions, the median is between these 2 positions. The median is computed as the average of the values at these positions.

$$\text{Median} = \frac{3 + 4}{2} = 3.5$$

(c) Quartiles

Approach: Arrange the data in ascending order. Items at the $1/4^{\text{th}}$ and $3/4^{\text{th}}$ positions are the first and third quartiles, respectively.

Example 5: Given 9 5 2 1 0 8 4 3. Determine the first and the third quartiles.

Solution

Arrange data in ascending order: 0 1 2 3 4 5 8 9

Number of data in the data set: $N = 8$

First Quartile

$$Q_1 = \frac{1}{4}(N+1)^{\text{th}} \text{ position} = \frac{1}{4}(8+1)^{\text{th}} \text{ position} = \frac{9^{\text{th}}}{4} = 2.25^{\text{th}} \text{ position}$$

Since item '1' is closer to this position, $Q_1 = 1$.

Third Quartile

$$Q_3 = \frac{3}{4}(N+1)^{\text{th}} \text{ position} = \frac{3}{4}(8+1)^{\text{th}} \text{ position} = \frac{27^{\text{th}}}{4} = 6.75^{\text{th}} \text{ position}$$

Since item '8' is closer to this position, $Q_3 = 8$.

Mode

- It is the item with the highest frequency / occurrence

Example 6: Given the following data set: 2, 3, -1, 2, 3, 2, 9, 0, -1, determine the mode.

Solution: Item with the highest occurrence is 2. Hence, mode = 2.

- If two items have the same highest frequency, then BOTH are selected as the mode and the distribution is called a BIMODAL DISTRIBUTION. If more than 2 modes are identified, then it is called a MULTIMODAL distribution.

Example 7: Given the following data set: 3, 7, -2, 3, 3, -2, 5, 8, -2. Determine the mode.

Solution

Items - 2 and 3 occur equally the most number of times. Hence, the mode = 3 and -2.

B. Computing Averages for Ungrouped Frequency Distribution**Arithmetic Mean**

- Formula: Mean, $\bar{x} = \frac{\sum f \cdot x}{\sum f}$

Example 8: Given the following data. Calculate the arithmetic mean.

Age	f
17	3
18	8
19	14
20	21
21	24
22	20

Solution:

Age (x)	f	fx
17	3	51
18	8	144
19	14	266
20	21	420
21	24	504
22	20	440
	$\Sigma f = 90$	$\Sigma fx = 1825$

$$\text{Mean, } \bar{x} = \frac{\sum fx}{\sum f} = \frac{1825}{90} = 20.28$$

Median and Quartiles

- Determine using the cumulative frequency table (cft).

Example 9: Given the following data, determine the median, first quartile and third quartile

x	3	4	5	6	7	8
f	16	24	27	29	16	3

Solution

X	cf
3	16
4	40
5	67
6	96
7	112
8	115

← (1st until 16th value)
 ← (17th until 40th value)
 ← (41st until 67th value)
 ← (68th until 96th value)
 ← (97th until 112th value)
 ← (113th until 115th value)

Determining the median

$$\text{Median} = \frac{1}{2}(N+1)^{\text{th}} \text{ position} = \frac{1}{2}(115+1)^{\text{th}} \text{ position} = 58^{\text{th}} \text{ position}$$

From the table above, the 58th position is between the range 41st until 67th position. The value 'x' at that position is 5. Hence, median = 5.

Determining the first quartile

$$Q_1 = \frac{3}{4}(N+1)^{\text{th}} \text{ position} = \frac{3}{4}(115+1)^{\text{th}} \text{ position} = 29^{\text{th}} \text{ position}$$

From the table above, the 29th position is between the range 17th until 40th position. The value 'x' at that position is 4. Hence, $Q_1 = 4$.

Determining the third quartile

$$Q_3 = \frac{3}{4}(N+1)^{\text{th}} \text{ position} = \frac{3}{4}(115+1)^{\text{th}} \text{ position} = 87^{\text{th}} \text{ position}$$

From the table above, the 87th position is between the range 68th until 96th position. The value 'x' at that position is 6. Hence, $Q_3 = 6$.

Mode

Example 10: For the data given below, determine the mode.

Height of student (cm)	f
155	3
156	7
157	10
158	15
159	16
160	9
161	2

Solution

The highest frequency is 16 and the corresponding height is 150 cm. So, the mode = 159 cm.

C. Computing Average for Grouped Frequency Distribution**a) Arithmetic Mean**

- Formula: Mean, $\bar{x} = \frac{\sum f \cdot x}{\sum f}$

Example 11:

A company selling a consumer product directly to retail outlet has collected the following information.

Average number of orders taken per month by individuals	Number of Salesmen
10 –19	3
20 –29	7
30 – 39	16
40 – 49	22
50 – 59	19
60 – 69	8
70 - 79	2

Solution

Make adjustment to the interval of the data so that it is in continuous form

Average Order, x	f	Mid- value	f.x
9.5 – 19.5	3	14.5	43.5
19.5 – 29.5	7	24.5	171.5
29.5 – 39.5	16	34.5	552
39.5 – 49.5	22	44.5	979
49.5 – 59.5	19	54.5	1035.5
59.5 – 69.5	8	64.5	516
69.5 – 79.5	2	74.5	149
	$\Sigma f = 77$		$\Sigma fx = 3346.5$

$$\text{Mean, } \bar{x} = \frac{\Sigma f.x}{\Sigma f} = \frac{3346.5}{77} = 44.76$$

b) Median and Quartiles

1. From the original data, create a cumulative frequency table (cft)
2. Using the cft, determine the median class, first quartile class and the third quartile class
3. The median can be computed using the following formulae

$$\text{Median} = L_M + \left[\frac{N}{2} - F_{M-1} \right] \frac{c}{f_m}$$

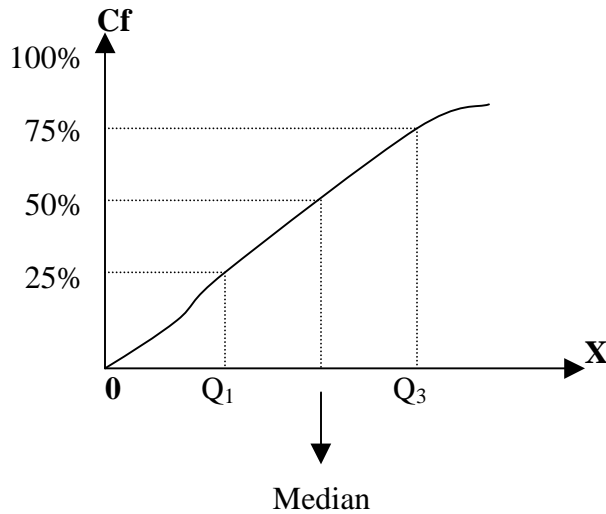
4. The first quartile can be computed using the formula below:

$$Q_1 = L_A + \left[\frac{N}{4} - F_{A-1} \right] \frac{c}{f_a}$$

5. The third quartile can be computed using the formula below:

$$Q_3 = L_B + \left[\frac{3N}{4} - F_{B-1} \right] \frac{c}{f_b}$$

Cumulative frequency curve (Ogive)



Example 12

Using the data from Example 11, estimate the median, first quartile and third quartile

Solution

First, create the ‘less than’ cumulative frequency

< x	f	cf	
< 19.5	3	3	
< 29.5	7	10	
< 39.5	16	26	← Q ₁ class
< 49.5	22	48	← Median Class
< 59.5	19	67	← Q ₃ class
< 69.5	8	75	
< 79.5	2	77	
	N = 77		

Computational Method

(i) Determining Median class

$$\text{Median} = \frac{1}{2} [N + 1]^{\text{th}} \text{ position} = \frac{1}{2} \times 78 = 39^{\text{th}} \text{ position}$$

So, the median class = ‘39.5 – 49.5’

(ii) Determining First Quartile Class

$$Q_1 = \frac{1}{4} \left[N + 1 \right]^{\text{th}} \text{ position} = \frac{1}{4} \times 78 = 19.5^{\text{th}} \text{ position}$$

So, the first quartile class = '29.5 -39.5'

(iii) Determining Third Quartile Class

$$Q_3 = \frac{3}{4} \left[N + 1 \right]^{\text{th}} \text{ position} = \frac{3}{4} \times 78 = 58.5^{\text{th}} \text{ position}$$

So, the third quartile class = '49.5 -59.5'

(i) Calculating Median

$$\text{Given, Median} = L_M + \left[\frac{N}{2} - F_{M-1} \right] \frac{C}{f_m}$$

where $L_M = 39.5$, $N = 77$, $f_m = 22$ and $C = 10$.

Applying the above value into the formula, we have

$$\begin{aligned} \text{Median} &= 39.5 + \left[\frac{77}{2} - 26 \right] \frac{10}{22} \\ &= 45.18 \end{aligned}$$

(ii) Calculating the first quartile

$$\text{Given, } Q_1 = L_a + \left[\frac{N}{4} - F_{A-1} \right] \frac{C}{f_a}$$

where $L_A = 29.5$, $N = 77$, $F_{A-1} = 10$, $f_a = 16$ and $C = 10$.

Applying the data above value into formulae, we have

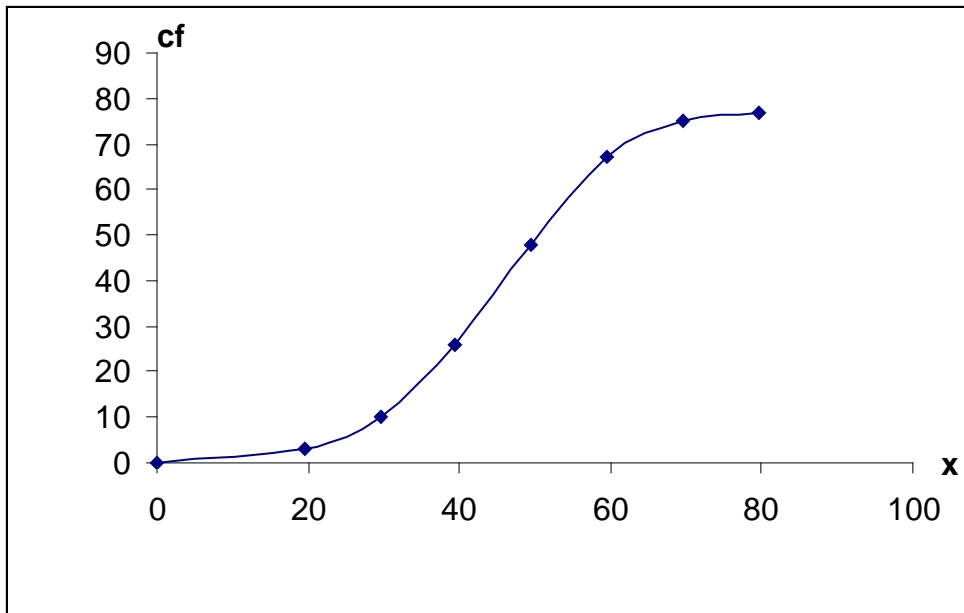
$$\begin{aligned} Q_1 &= 29.5 + \left[\frac{77}{4} - 10 \right] \frac{10}{16} \\ &= 35.28 \end{aligned}$$

(iii) Calculating the third quartile

Given, $Q_1 = L_B + \left[\frac{3N}{4} - F_{B-1} \right] \frac{c}{f_b}$ where $L_B = 49.5$, $N = 77$, $F_{B-1} = 48$, $f_b = 19$ and $C = 10$

Applying the above value into the formulae, we have

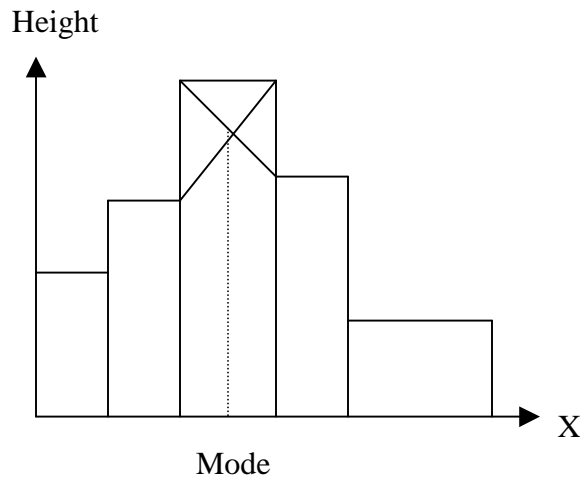
$$Q_3 = 49.5 + \left[\frac{231}{4} - 48 \right] \frac{10}{19} = \underline{\underline{54.63}}$$

Using Ogive**c) Mode****Computational Method**

1. From the given data, determine the modal class
2. Determine the mode using the following formula

$$\text{Mode} = L_m + \left[\frac{f_m - f_{m-1}}{2f_m - f_{m-1} - f_{m+1}} \right] C$$

From Histogram



Example 13

A company selling a consumer product directly to retail outlets has collected the following information.

Average number of orders taken per month by an individual	Number of salesmen
10 –19	3
20 – 29	7
30 –39	16
40 –49	22
50 –59	19
60 –69	8
70 -79	2

Determine the mode for the average orders

Solution

Given,

Average Order, x	<i>f</i>
9.5 – 19.5	3
19.5 – 29.5	7
29.5 – 39.5	16
39.5 – 49.5	22
49.5 – 59.5	19
59.5 – 69.5	8
69.5 – 79.5	2
	$\Sigma f = 77$

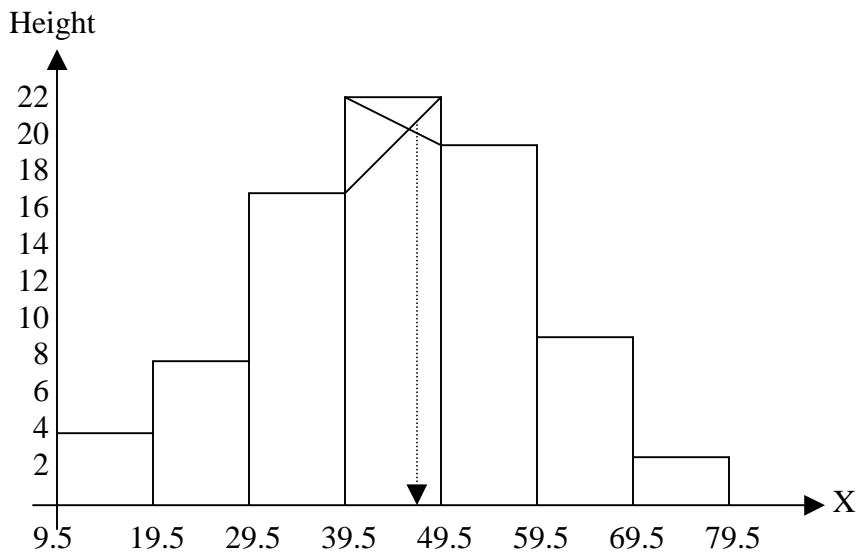
← Mode Class
Computational Method

$$\text{Given, Mode} = L_m + \left[\frac{f_m - f_{m-1}}{2f_m - f_m - f_{m-1}} \right] C$$

where $L_m = 39.5$, $f_m = 22$, $f_{m-1} = 16$, $f_{m+1} = 19$ and $C = 10$.

From Histogram

Using the data given, draw a histogram.



From the histogram, the mode $\cong 46$

Learning Outcomes

- Students should be able to use summary statistics to describe collection of data.
- Students should be able to use the mean, median, quartiles and percentile to describe how data 'bunch up'.

Basic Reading

1. Saravanan Kullandavelli (1994) LCCI Business Statistics; 5th ed. Malaysia; Stamford College Group Publishing.
2. A Francis (1995) Business Mathematics and Statistics; 4th ed. London DP Publications Ltd.

Revision Questions

- For the following set of data items, determine the mean, median, first quartile, third quartile and mode.
 - 3, 7, -9, 9, -9, 3, 8
 - 6, 6, 6, 7, -6, 4, 5, 8, 19
 - 3, 3, 3, 3, 4, 4, 5, 6, 6, 6, 6, 7, -2
- Given the height of 100 students, determine the mean, median, first quartile, third quartile and mode

Height (in cm)	Number of Students
150	8
151	13
152	12
153	7
154	30
155	19
156	11

- In a certain organisation, employees were categorised according to their ages as shown below:

Age Group	No. of Employees
15 – 19	66
20 – 24	65
25 – 29	56
30 – 34	50
35 – 39	42
40 – 44	37
45 – 49	35
50 – 54	30
55 – 59	24
60 and over	22

Calculate the mean, median, mode and quartiles of ages of employees in the industry.