CHAPTER ELEVEN : HYPOTHESIS TESTING

Introduction

- Hypothesis testing can be defined as 'methods used to establish whether or not the value of a known population parameter has changed or not'. For example, if it is said that "the mean income per annum of all the engineers in the country is RMY", we can then ask the question "Is the mean income per annum of all engineers is still RMY or has it changed?" We can then go on to conduct a hypothesis test to specify whether the true mean income has increased (greater than RMY) or decreased (less than RMY) or merely different from the hypothesized value (RMY).

- The following statements are examples of hypothesis test which can be conducted to verify their validity:
  a. The mean annual household income is at least RM12,000
  b. The mean life of a car battery is not more than 40 months.
  c. A manufacturer of a new tooth paste claims that it reduces the pain caused by sensitive teeth in 90% of the cases.
  d. A television executive claims that the majority of teenagers are in favour of sport shows on television.

Components of a hypothesis test

The key components that make up a hypothesis test are:

i. Null and Alternative hypotheses
ii. One-sided and two-sided tests
iii. Significance level
iv. Test statistics
v. Type I and Type II errors
vi. Acceptance and Rejection Regions

Null hypothesis

The null hypothesis is denoted by the symbol H_0. It specifies the value of the population parameter to be tested in a hypothesis test. The word "null" means "no difference", indicating that there is no difference between the parameter value specified in the null hypothesis and the actual value of the population parameter. The null hypothesis is tested directly in the sense that the final conclusion will be either rejection of H_0 or failure to reject H_0.

Alternative hypothesis

The alternative hypothesis is denoted by the symbol H_A or H_1. It is the statement that must be true if the null hypothesis is rejected.

One-sided test and Two-sided test

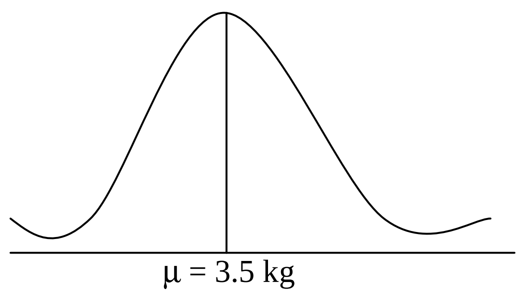
The first step in setting up a hypothesis test is to decide what the null hypothesis and the alternative hypothesis should be. Let's analyse the following statement:

"A supplier claims that the mean weight of materials supplied by him is 3.5kg."

The claim that was made by the supplier needs to be tested. The hypothesis that will be used to test the validity of the statement is the Null hypothesis. Since the supplier has claimed that mean weight of materials is exactly 3.5 kg, the null hypothesis will be written in such a way as to support the statement. The null hypothesis can be written as:

H_0: $\mu = 3.5$ kg.

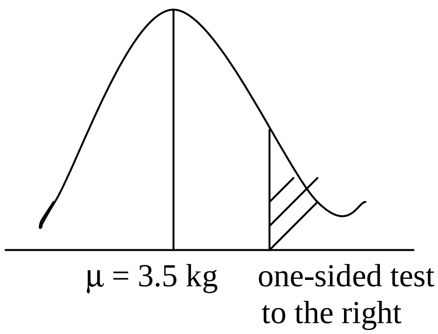
On a normal distribution curve, the 'μ" value is the population parameter and is placed at the center of the curve.

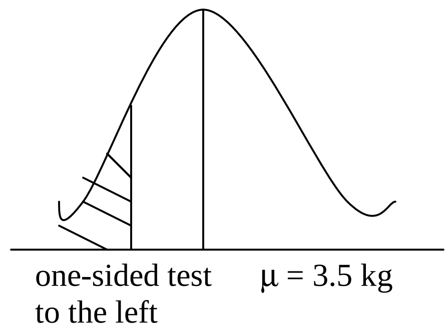


Since the null hypothesis (the suppliers' claim) might not be true, we have to accept the alternative hypothesis which might be any ONE of the following:

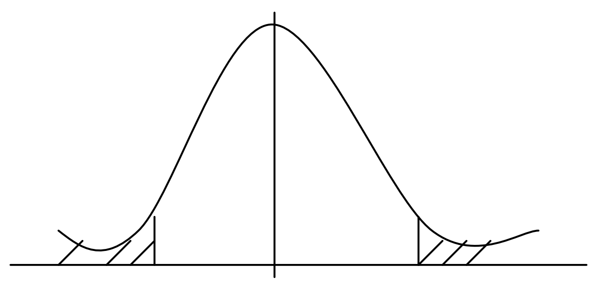
	H_A: $\mu > 3.5$ kg	(1)	indicates the mean weight is actually greater than 3.5kg
OR	H_A: $\mu < 3.5$ kg	(2)	indicates the mean weight is actually less than 3.5kg
OR	H_A: $\mu \neq 3.5$ kg	(3)	indicates the mean weight is NOT equal to 3.5 kg.

If the alternative hypothesis is either of the first two (1 or 2) then the statistical test would be referred to as a **one-tailed test** (one-sided test). A one-tailed statistical test is one that locates the rejection region in only one tail of the sampling distribution of the test statistic. On a normal distribution curve, both the options ($\mu > 3.5$ kg or $\mu < 3.5$ kg) would be represented by the shaded area shown below:





If on the other hand the alternative hypothesis is of case (3), then the statistical test would be referred to as a **two-tailed test** (two-sided test). A two-tailed test is one that locates the rejection region in both tails of the sampling distribution of the test statistic. On a normal distribution curve, this option would be represented as follows:



The key problem in a hypothesis test is to decide when to use a one-sided test and when to use a two-sided test. In deciding which test to use, first we have to know two key characteristics of a hypothesis test.

1. In conducting a formal statistical hypothesis test, we are always testing the null hypothesis, whether it corresponds to the original claim or not. Sometimes, the null hypothesis corresponds to the original claim and sometimes it corresponds to the opposite of the original claim. Since we always test the null hypothesis, we will be testing the original claim in some cases and the opposite of the original claim in other cases.
2. The null hypothesis always has a statement of **equality** in it. Hence, the statement of a null hypothesis can be of three types:

H_0: $\mu = 28.5$ (1)
OR H_0: $\mu \geq 28.5$ (2)
OR H_0: $\mu \leq 28.5$ (3)

For each of the statements given (1, 2 and 3), note that an equality is always included.

- Now, if the null hypothesis statement is of case (1), the alternative hypothesis would be:

  H_0: $\mu = 28.5$
  H_A: $\mu \neq 28.5$ ← A Two-sided test

- Now, if the null hypothesis statement is of case (2), the alternative hypothesis would be:

  H_0: $\mu \geq 28.5$
  H_A: $\mu < 28.5$ ← A One-sided test to the left

- Now, if the null hypothesis statement is of case (3), the alternative hypothesis would be:

  H_0: $\mu \leq 28.5$
  H_A: $\mu > 28.5$ ← A One-sided test to the right

Note that the alternative hypothesis does **NOT** have a statement of equality in it.

Type I and Type II error

Whenever a hypothesis test is conducted, it is always possible that the decision reached will be incorrect. This is because partial information, obtained from the sample, is used to draw conclusions about the entire population. In a hypothesis test, there are four possible outcomes. Two of the outcomes will lead to an incorrect decision. The four possible outcomes are shown below:

Decision	**Actual decision that should be taken**
Accept H_0	Correct decision: H_0 is true and should be accepted
Accept H_0	Wrong decision: H_0 is false and should be rejected. We call this a Type II error
Reject H_0	Correct decision: H_0 is false and should be rejected
Reject H_0	Wrong decision: H_0 is true and should not be rejected. We call this a Type I error

Type I error : We reject the null hypothesis when in fact it is true.
Type II error : We accept the null hypothesis when in fact it is false.

Significance Level

The probability of making Type I error is actually the probability of rejecting the null hypothesis when in fact it is true. This probability is called the significance level of the hypothesis test and is denoted by the symbol α.

The common significance level used in a hypothesis test is 1% (0.01) or 5% (0.05).

The critical value, C, is dependent on the significance level that is used and the type of test (one-tailed or two-tailed). The table below shows the significance level used, the type of test and the critical value, C. Since the critical value must be converted into a normal statistical value, it is denoted as Z_α for a one-tailed test and as $Z_{\alpha/2}$ foe a two-tailed test. The critical value can be determined for any significance level from a normal distribution table.

Significance level, α	**Type of Test**	**Critical value**
1%	One-tailed (to the left)	-2.33 ($-Z_{0.01}$)
1%	One-tailed (to the right)	+2.33 ($+Z_{0.01}$)
1%	Two-tailed	±2.58 ($\pm Z_{0.005}$)
5%	One-tailed (to the left)	-1.65 ($-Z_{0.05}$)
5%	One-tailed (to the right)	+1.65 ($+Z_{0.05}$)
5%	Two-tailed	±1.96 ($\pm Z_{0.025}$)
10%	One-tailed (to the left)	-1.28 ($-Z_{0.1}$)
10%	One-tailed (to the right)	+1.28 ($+Z_{0.1}$)
10%	Two-tailed	±1.65 ($\pm Z_{0.05}$)

Steps in conducting a hypothesis test

The following steps can be followed in sequence to conduct a hypothesis test:

1. Identify the specific claim or hypothesis to be tested. Determine whether the test is a one-tailed or two-tailed test.

2. Write the statements for the null hypothesis and the alternative hypothesis. Let H_0 be the statement that contains equality and H_A the other.

3. Determine the significance level, α.

4. Determine the test statistic and the critical value(s).

5. Draw a graph (Normal curve) and include the test statistic, critical value(s), acceptance and rejection regions.

6. Reject H_0 if the test statistic is in the rejection region. Otherwise, accept H_0. Formally, re-state the decision.

Hypothesis Test on Mean

Sample Mean Test for Large Sample (n≥30)

The test statistic, Z_c, for a sampling distribution of the mean is given by:

$$Z_C = \frac{\bar{x} - \mu}{SE} \quad \text{where} \quad SE = \frac{\sigma}{\sqrt{n}}$$

Example 1

The manufacturer of a popular soft drink claims that each bottle of soft drink supplied by the company contains 230 ml. An inspector from the consumer representative organization wishes to validate the claim. He has collected a random sample of 70 bottles and found that the mean contents to be 228 ml with a standard deviation of 2.5 ml. Verify the manufacturer's claim at the 5% significance level.

Solution

$H_0 : \mu = 230$ ml
$H_A : \mu \neq 230$ ml ◄——— A Two-sided test

At 5% significance level, the critical value would be ± 1.96

Test statistic, $Z_c = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{228 - 230}{\frac{2.5}{\sqrt{70}}}$

$= \underline{-6.67}$

∴The test statistic Z_c is less than the critical value -1.96, that is, the test statistic is in the rejection region. We reject the null hypothesis. Hence, at the 5% significance level, we reject the manufacturer's claim.

Example 2

A paint manufacturer claims that the average drying time for its new paint is less than 90 minutes. To test the claim, the drying times are obtained for 50 randomly selected paints. The sample mean and the standard deviation for the sample were found to be 88 minutes and 8 minutes, respectively. At the 1% significance level, test the claim made by the manufacturer.

Solution

H_0: $\mu \geq 90$
H_A: $\mu < 90$ ← A one-sided test to the left

At 1% significance level, the critical value would be –2.33.

Test statistic, $Z_c = \dfrac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \dfrac{88-90}{\frac{8}{\sqrt{50}}}$

$= \underline{-1.77}$

$\therefore$ The test statistic Z_c is greater than the critical value -2.33, that is, the test statistic is in the acceptance region. At significance level of 1%, we accept the null hypothesis. But the statement of the null hypothesis does not support the supplier's claim. That means, at 1% significance level, we reject the claim made by the supplier.

Hypothesis Test on Proportion

The test statistic, Z_c, for a sampling distribution of the proportion is given by:

$$Z_C = \frac{p-\pi}{SE} \quad \text{where} \quad SE = \sqrt{\frac{\pi(1-\pi)}{n}}$$

Example 3

A travel agency wishes to know whether their customers are satisfied with their services. A random sample of 300 of their customers was interviewed and 180 said they are satisfied. Does the sample result support the hypothesis that at least 80% of their customers are satisfied with their services? Test at 1% significance level.

Solution

$H_0: \Pi \geq 0.8$
$H_A: \Pi < 0.8$ ⟵ A one-sided test to the left

At 1% significance level, the critical value would be –2.33.

$$\text{Test statistic, } Z_C = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.6-0.8}{\sqrt{\frac{0.8(1-0.8)}{300}}}$$
$$= \underline{-8.7}$$

∴ The test statistic value is less than the critical value, that is, the test statistic is in the rejection region. We reject the null hypothesis. At 1% significance level, we reject the claim made by the travel agency that at least 80% of their customers are satisfied with their services.

Example 4

An auditor claims that 10% of invoices for a company are incorrect. To test this claim a random sample of 100 invoices are checked and 12 are found to be incorrect. Test, at the 5% significance level, whether the sample evidence supports the auditor's claim.

Solution

$H_0: \Pi = 0.1$
$H_A: \Pi \neq 0.1$ ⟵ A two-sided test

At 5% significance level, the critical value would be ± 1.96.

$$\text{Test statistic, } Z_C = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.12-0.1}{\sqrt{\frac{0.1(1-0.1)}{100}}}$$
$$= \underline{0.67}$$

∴ The test statistic value is in between the critical values, that is, the test statistic is in the acceptance region. We accept the null hypothesis. At 5% significance level, we can conclude that there is no evidence to support the auditor's claim.

Learning Outcomes

- Students should be able to understand the concepts of hypothesis testing.
- Students should be able to identify null hypothesis and alternative hypothesis.
- Students should be able to understand the concepts of Type I error and Type II error.
- Students should be able to calculate test statistics.

Basic Reading

1. Saravanan Kullandavelli (1994) LCCI Business Statistics; 5th ed. Malaysia; Stamford College Group Publishing.
2. A Francis (1995) Business Mathematics and Statistics; 4th ed. London DP Publications Ltd.

Revision Questions

1. Briefly explain the following terms:

a.	hypothesis test	b.	null hypothesis
c.	alternative hypothesis	d.	one-tailed test
e.	two-tailed test	f.	significance level
g.	critical value	h.	acceptance region

2. It is required to test the hypothesis that a majority of administration offices in a certain city use computers. A random sample of 100 such offices were taken and it was found that 52% of the sample use computers in their work. Conduct a hypothesis test with a 5% significance level.

3. The management of a fast-food chain restaurant claims that at least 90% of their customers are aged below 25 years old. A random sample of 300 customers was taken and 240 are below the age 25 years old.

a) Test whether the management's claim is validated at a 5% level of significance.
b) If the sample size was increased to 500, what is the maximum number of customers that should be below the age of 25 years old which would be acceptable under the management's claim?